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Condensed gas evaporation into the atmosphere

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Abstract—The rate of evaporation of liquid oxygen depends upon the density distribution of gaseous oxygen in the vicinity of the phase interface. The paper represents theoretical and experimental study of the problem. An analytical self-similar solution is obtained for the system of unsteady one-dimensional equations describing the flow of heat conducting mixture of mutually diffusing gases above the thermally destructing surface. The rate of evaporation, concentration and temperature profiles for various times are determined theoretically and compared with the experimental data.

1. INTRODUCTION

ONE OF the current central problems of interphase heat and mass transfer is evaporation of liquid contacting a multi-component gas mixture. Evaporation of a condensed gas from a free surface into the atmosphere is not only of theoretical interest but also of practical importance.

For determining the mass velocity of liquid evaporation, it is necessary to study the unsteady-state processes of the formation of a 'condensed component vapours–air' mixture with account for the coupled heat transfer and diffusion processes in the gaseous and condensed phases, and also for the conservation laws and evaporation conditions on the moving phase interface. The solution of this problem in a rigorous formulation presents great difficulties. This explains the upswing of interest in the development of approximate techniques based on simplified models of condensed gas evaporation into the atmosphere which rather accurately describe some specific cases of the process and substantially shorten the amount of computation. It should be noted that the simplest one-dimensional stationary statement of the problem does not have a solution. Invoking a number of additional assumptions, some of the authors have managed to obtain the solution of the problem for a two-dimensional stationary case [1] and for a one-dimensional unstationary case [2–8]. Their common weakness is the hypothesized constant density of the mixture of gases.

A characteristic feature of the process of condensed gas evaporation is the existence of large temperature and concentration gradients of the evaporating component in the gas. With the pressure being constant, this means that the density of the mixture of gases ρ is a function of temperature T and of the evaporating component concentration Y: $\rho = \rho(Y, T)$. The assumption that $\rho =$ const. only qualitatively contradicts the physics of the phenomenon for vapours with a very small molecular weight (hydrogen, helium).

The objective of this study was to develop a method for calculating unsteady-state process of condensed gas evaporation into the atmosphere taking account of the change in the density over the phase interface.

2. STATEMENT OF THE PROBLEM

The practically important dimensions of the displacement regions and the characteristic times of the progress of processes in evaporation of large volumes of condensed gases into the atmosphere are such that the time and space dependence of pressure can be neglected. The boundary conditions in the gas may be transferred to infinity. To be explicit, it will be assumed that the mixture of gases is two-component. The effect of external body forces is neglected.

A system of coordinates will be considered in which the vertical axis $0y$ is connected with the phase interface ($y = 0$). The semi-space $y > 0$ at the initial time $t = 0$ is occupied by air whereas the semi-space $y < 0$ is occupied by a condensed gas.

The equations of mass, mixture energy, and evaporating component mass conservation, augmented with the equation of state, are of the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial y} = 0,$$

$$\frac{\partial \rho Y}{\partial t} + \frac{\partial \rho v Y}{\partial y} = \left(\rho D \frac{\partial Y}{\partial y}\right),$$

$$c_{\mathrm{p}}\left(\frac{\partial \rho T}{\partial t} + \frac{\partial \rho v T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right),$$

$$p = \rho R T\left(\frac{Y}{m_1} + \frac{1-Y}{m_2}\right). \quad (1)$$

Equations (1), which describe the distribution of the gas phase parameters, are being solved together

NOMENCLATURE

c_p	specific heat at constant pressure
D	coefficient of binary diffusion
erf s	probability integral, $(2/\sqrt{\pi})\int_0^s \exp(-s^2/2)\,ds$
f	self-similar velocity function
g	dimensionless temperature of liquid
h_1	specific heat of phase
Le	Lewis number, $\lambda/(\rho c_p D)$
m_i	molar mass of the ith component
$\dot{m}$	mass flux through the phase interface
p	pressure
R	universal gas constant
Sc	Schmidt number, $\mu/\rho D$
T	temperature
t	time
v	velocity
Y	mass concentration
y	spatial coordinate
z, s	new dependent variables.

Greek symbols

δ	ratio of initial gas and liquid densities
η	density function
λ	thermal conductivity
μ	viscosity
ξ	self-similar variable
ρ	density
φ	concentration function.

Subscripts

H	liquid initial parameters
$i = 1$	evaporating component
l	current values of liquid phase parameters
0	gas initial parameters
w	gas parameters near a phase interface.

with the equation of energy in a condensed gas. In the chosen coordinate system the latter equation has the form

$$\rho_l \frac{\partial T_l}{\partial t} + v_l \rho_l \frac{\partial T_0}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\lambda_l}{c_{pl}}\frac{\partial T_l}{\partial y}\right), \tag{2}$$

where v and p are the velocity and pressure of the mixture of gases; μ, D and λ are the coefficients of viscosity, diffusion and heat conduction, c_p is the specific heat, R the universal gas constant, m_1 and m_2 the molecular masses of components, l the index of the condensed gas parameters.

The initial and boundary conditions of equations (1) and (2) are as follows:

$$\begin{aligned} t = 0 \quad y > 0 \quad T = T_0, \quad Y = Y_0, \\ v = 0 \quad y < 0 \quad T_l = T_H \\ y \to +\infty \quad Y = Y_0, \quad T = T_0 \\ y \to -\infty \quad T_l = T_H. \end{aligned} \tag{3}$$

The boundary conditions on the phase interface ($y = 0$) follow from the mass conservation equation for the mass fluxes, ith component, energy conservation equation and from the condition of equilibrium evaporation. In the chosen coordinate system they have the form

$$\begin{aligned} \rho_w v_w = \rho_l v_l = \dot{m} \\ \dot{m} Y_w = \dot{m} Y_w - (\rho D)_w \left(\frac{\partial Y}{\partial y}\right)_w \\ \dot{m} h_L - \left(\lambda \frac{\partial T}{\partial y}\right)_w = \left(\lambda_l \frac{\partial T}{\partial y}\right)_{lw} \\ T_w = T(p_{1w}) \end{aligned} \tag{4}$$

where h_L is the latent heat of evaporation, p_{1w} is the partial pressure of the evaporating component, w is the subscript relating to the phase interface parameters.

3. SELF-SIMILAR STATEMENT OF THE PROBLEM

The further discussion will be restricted to the case in which the condition $\rho\mu =$ const. is fulfilled. For a perfect gas this condition means proportionality of the viscosity coefficient to temperature: $\mu \sim T$. For a mixture of perfect gases such an assumption is admissible if ρ and μ are weak functions of the mixture composition, i.e. the molecular masses and other characteristic physical–chemical properties of the evaporating component and air are almost similar. This takes place in such practically important cases as evaporation of a liquified oxygen or nitrogen. In this case, when the Schmidt ($Sc = \mu/\rho D$) and Lewis ($Le = \lambda/\rho D c_p$) numbers are constant, the set of equations (1) and (2) with initial and boundary conditions (3) admits a self-similar solution. The independent variable

$$\xi = \frac{y}{2\sqrt{(\bar{D}_0 t)}} \tag{5}$$

and dependent variables

$$\varphi(\xi) = \frac{Y}{Y_w}, \quad \eta(\xi) = \frac{\rho}{\rho_0}, \quad \rho v = \rho_0 f(\xi)\frac{\sqrt{(D_0)}}{2\sqrt{(t)}};$$

$$\psi(\xi) = \frac{T - T_0}{T_w - T_0}; \quad g(\xi) = \frac{T_l - T_H}{T_w - T_H} \tag{6}$$

are introduced where 0 is the subscript referring to the gas parameters at infinity.

The introduction of self-similar variables transforms the set of equations (1) and (2) with initial and boundary conditions (3) into the following set of equations

$$-2\eta'(\xi)\xi+f'(\xi)=0 \tag{7}$$

$$\frac{\varphi''(\xi)}{\eta(\xi)}-\frac{\eta'(\xi)\psi'(\xi)}{\eta^2(\xi)}=\psi'(\xi)(f(\xi)-2\xi\eta(\xi)) \tag{8}$$

$$Le\left(\frac{\psi''(\xi)}{\eta(\xi)}-\frac{\psi'(\xi)\eta'(\xi)}{\eta^2(\xi)}\right)=\psi'(\xi)(f(\xi)-2\xi\eta(\xi)) \tag{9}$$

$$Le_l\, g''(\xi)+g'(\xi)(2\xi-\delta f(0))=0 \tag{10}$$

$$\frac{p}{(T_w-T_0)\rho_0 R}=\eta(\xi)\left(\psi(\xi)+\frac{T_0}{T_w-T_0}\right)(AY_w\varphi(\xi)+B),$$

where

$$Le_l=\frac{\lambda_l}{\rho_l c_{pl} D_0},\quad \delta=\frac{\rho_0}{\rho_l},\quad B=\frac{1}{m_2},\quad A=\frac{1}{m_1}-B. \tag{11}$$

The boundary conditions of the system in dimensionless parameters have the form

$$\xi\to+\infty,\quad \varphi=\frac{Y_0}{Y_w},\quad \psi=0 \tag{12}$$

$$\xi\to-\infty,\quad g=0. \tag{13}$$

At the phase interface the dimensionless variables take on the values

$$\xi=0,\quad \varphi=1,\quad \psi=1,\quad g=1,$$

$$f=f(0),\quad \eta=\eta(0)=\frac{\rho_w}{\rho_0}. \tag{14}$$

The set of equations (7)–(11) subject to boundary conditions (12)–(14) is closed with respect to the unknown functions $\varphi(\xi)$, $\psi(\xi)$, $g(\xi)$, $f(\xi)$, and $\eta(\xi)$. At the same time, boundary conditions (12) and (14) and also formulae (6), which connect dimensionless and dimensional variables, contain the unknown values of the functions on the phase interface. To determine these unknown values of Y_w, T_w, $f(0)$, and $\eta(0)$, use will be made of relationships on the phase interface (4) and of the state equation which, with account for the fact that equations (5) and (6) have become nondimensional, assume on the form

$$\frac{h_L}{c_p(T_w-T_0)}f(0)=Le\frac{\psi'(0)}{\eta(0)}-\frac{Le}{\eta(0)}\frac{\lambda_l}{\lambda}\frac{T_w-T_H}{T_w-T_0}g'(0)$$

$$f(0)\frac{Y_w-1}{Y_w}=\varphi'(0)\eta(0)$$

$$1+\frac{m_1}{m_2}\left(\frac{1-Y_w}{Y_w}\right)=\exp\left[A_1\left(\frac{1}{T_w}-\frac{1}{T_k}\right)\right]$$

$$\frac{p}{R\rho_0 T_w}=\eta(0)(AY_w+B), \tag{15}$$

where A_1, T_k are the constants of the equilibrium evaporation curve.

Note that besides the unknown values Y_w, T_w, $f(0)$ and $\eta(0)$, equation (15) also involves the boundary values of the derived functions $\varphi'(0)$, $\psi'(0)$ and $g'(0)$ which must be determined from the set of equations (7)–(11). Thus, the set of equations (7)–(11) with boundary conditions (12)–(14) should be solved together with system (15).

4. SOLUTION OF EQUATIONS

For solving this problem, equation (7) will be rewritten as

$$(f-2\xi\eta)'=-2\eta. \tag{16}$$

Taking into account the equality

$$\frac{\varphi''}{\eta}-\frac{\eta'\varphi'}{\eta^2}=\left(\frac{\varphi'}{\eta}\right)'$$

and equation (16), transform equation (8) as

$$\left(\frac{\varphi'}{\eta}\right)=\frac{\varphi'(f-2\xi\eta)(f-2\xi\eta)'}{-2\eta}. \tag{17}$$

With a further replacement of unknowns

$$z=\frac{\varphi'}{\eta};\quad s=f-2\xi\eta, \tag{18}$$

equation (17) yields the following equation

$$z'=-\frac{z}{2}ss',$$

the integration of which gives

$$z=c_1\,e^{-s^2/4}. \tag{19}$$

The inverse replacement of $z=\varphi'/\eta$ results in

$$\varphi'=c_1\eta\,e^{-s^2/4}. \tag{20}$$

Since it follows from equation (16) that

$$\eta=-\frac{(f-2\xi\eta)'}{2}$$

or, with equation (18) taken into account, that

$$\eta=\frac{s'}{2}$$

then, substituting the latter equation into equation (20) gives

$$\varphi'(\xi)=-\tfrac{1}{2}c_1\exp\left(-\frac{s^2(\xi)}{4}\right)s'(\xi). \tag{21}$$

The integration of equation (21) and determination of the integration constants from conditions (12) and (14) yields

$$\varphi(\xi) = \frac{1+\operatorname{erf}\left(\frac{s}{2}\right)+\frac{Y_0}{Y_w}\left[\operatorname{erf}\left(\frac{f(0)}{2}\right)-\operatorname{erf}\left(\frac{s}{2}\right)\right]}{1+\operatorname{erf}\left(\frac{f(0)}{2}\right)}. \tag{22}$$

In this case the expression for $\varphi'(0)$ required for solving the set of equations (15) on the phase interphase has the form:

$$\varphi'(0) = \eta(0)\exp\left(-\left(\frac{f(0)}{2}\right)^2\right) \times \frac{2}{\sqrt{\pi}}\,\frac{1-Y_0/Y_w}{1+\operatorname{erf}\left(\frac{f(0)}{2}\right)}. \tag{23}$$

Proceeding in a completely analogous way with equation (9) we obtain

$$\psi(\xi) = \frac{1+\operatorname{erf}\left(\frac{s}{2\sqrt{(Le)}}\right)}{1+\operatorname{erf}\left(\frac{f(0)}{2\sqrt{(Le)}}\right)}, \tag{24}$$

$$\psi'(0) = -\eta(0)\exp\left(-\frac{f^2(0)}{4\,Le}\right) \times \frac{2}{\sqrt{(\pi\,Le)}}\,\frac{1}{1+\operatorname{erf}\left(\frac{f(0)}{2\sqrt{(Le)}}\right)}. \tag{25}$$

Integration of equation (10) gives

$$g'(\xi) = c_2\exp\left(-\frac{(2\xi-\delta(0))^2}{4\,Le_l}\right). \tag{26}$$

Having integrated equation (26) with account for boundary conditions (13) and (14), we obtain the solution of equation (10):

$$g(\xi) = \frac{1-\operatorname{erf}\left(\frac{2\xi-\delta f(0)}{2\sqrt{(Le_l)}}\right)}{1-\operatorname{erf}\left(\frac{\delta f(0)}{2\sqrt{(Le_l)}}\right)} \tag{27}$$

in this case, the expression for $g'(0)$ has the form

$$g'(0) = \frac{2}{\sqrt{(\pi\,Le_l)}}\,\frac{1}{1-\operatorname{erf}\left(\frac{\delta f(0)}{2\sqrt{(Le_l)}}\right)} \times\exp\left(-\frac{(\delta f(0))^2}{4\,Le_l}\right). \tag{28}$$

After the substitution of equations (22) and (24) for $\varphi(\xi)$ and $\psi(\xi)$ into equation (11), the latter takes on the form

$$\eta(\xi) = G(s) \tag{29}$$

where

$$G(s) = \frac{\rho}{\rho_0 R}\left\{\left[(T_w-T_0)\frac{\left(1+\operatorname{erf}\left(\frac{s}{2\sqrt{(Le)}}\right)\right)}{\left(1+\operatorname{erf}\left(\frac{f(0)}{2\sqrt{(Le)}}\right)\right)}+T_0\right]\right.$$

$$\left.\times\left[A\,Y_w\frac{1+\operatorname{erf}\left(\frac{s}{2}\right)+\frac{T_0}{Y_w}\left(\operatorname{erf}\left(\frac{f(0)}{2}\right)-\operatorname{erf}\left(\frac{s}{2}\right)\right)}{1+\operatorname{erf}\left(\frac{f(0)}{2}\right)}+B\right]\right\}.$$

According to equations (16), (18) and (29),

$$\frac{\mathrm{d}s}{\mathrm{d}\xi} = -2G(s). \tag{30}$$

Thus, the problem has been reduced to the solution of an ordinary differential equation (30) with boundary condition $s(0) = f(0)$.

The value of $f(0)$ and also of $\eta(0)$, Y_w and T_w can be found by solving the set of equations (15) with account for the obtained expressions (23), (25) and (28) for $\varphi'(0)$, $\psi'(0)$ and $g'(0)$. Substituting the resulting numerical solution of $s(\xi)$ from equation (30) into equations (22), (24), (27) and (29) yields the solutions for the functions $\varphi(\xi)$, $\psi(\xi)$, $g(\xi)$ and $\eta(\xi)$. Knowing $s(\xi)$ and $\eta(\xi)$, it is possible to determine $f(\xi)$ from equation (18).

The transition from the found self-similar functions to physical variables is prescribed by the following relations

$$Y = Y_w\varphi(\xi),\quad v = \frac{\rho_0}{\rho}f(\xi)\frac{\sqrt{(D_0)}}{2\sqrt{(t)}}$$

$$\rho = \rho_0\eta(\xi),\quad T = T_0+(T_w-T_0)\psi(\xi),$$

$$T_l = T_H+(T_0-T_H)g(\xi).$$

5. RESULTS OF CALCULATIONS AND THEIR COMPARISON WITH EXPERIMENTAL DATA

The predicted concentration and temperature distributions in the liquid oxygen evaporation zone were compared with the results of an experiment conducted on a set-up described elsewhere [9]. The experimental investigation was undertaken to study the change in concentration and temperature fields over oxygen evaporating in a vessel at different time instants and on different levels.

The results of numerical calculations and experimental data are presented in Figs. 1–5 as plots of oxygen concentration and mixture temperature vs the coordinate y and t. In Fig. 1 curves 1, 2 and 3 correspond to the oxygen concentration distribution above the evaporation surface for the time instants $t_1 = 60$, $t_2 = 300$, $t_3 = 6000$ s. The experimental concentration data in Fig. 1 are designated as '○' for t_1,

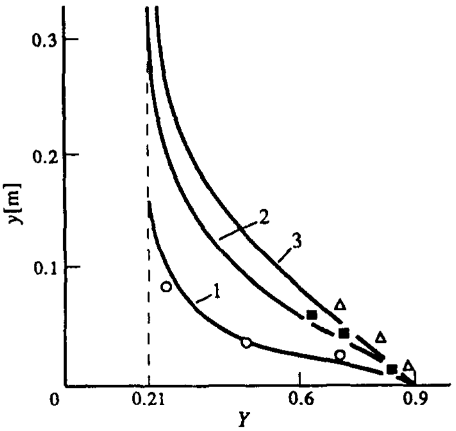

FIG. 1. Distribution of oxygen concentration above the evaporation surface at different time instants.

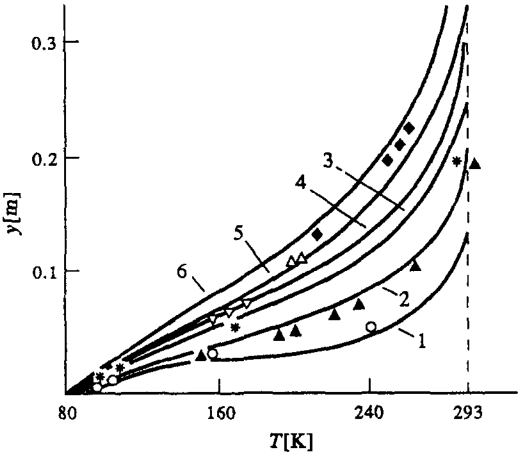

FIG. 2. Dependence of temperature on the height above the evaporation surface.

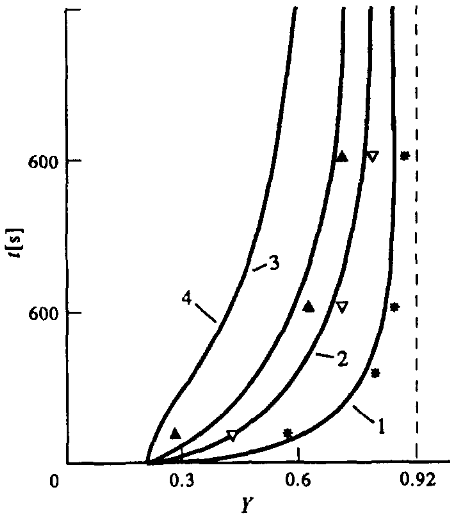

FIG. 3. Variation of oxygen concentration in time at different heights.

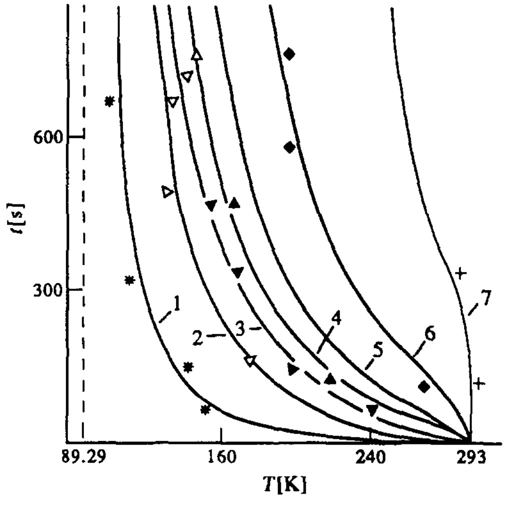

FIG. 4. Dependence of temperature on time at different heights.

■ for t_2, △ for t_3, respectively. Curves 1–6 in Fig. 2 represent theoretical relations of the mixture temperature vs height. These correspond to the times $t_1 = 60$, $t_2 = 180$, $t_3 = 330$, $t_4 = 420$, $T_5 = 600$, $t_6 = 900$ s. The experimental data for the indicated time instants are labelled as: ○, ▲, *, ▽, △, ◆, respectively.

Curves 1–4 in Fig. 3 determine the change in the oxygen concentration with time at the height $y_1 = 0.02$, $y_2 = 0.04$, $y_3 = 0.06$ and $y_4 = 0.1$ m. The symbols *, ▽, ▲ correspond to the measured values of concentrations at the levels y_1, y_2 and y_3.

In Fig. 4, curves 1–7 and the symbols *, ▽, ▼, △, ○, ◆ and + depict the predicted and experimental dependences of temperature on time for the levels $y_1 = 0.02, y_2 = 0.04, y_3 = 0.05, y_4 = 0.06, y_5 = 0.07$, $y_6 = 0.1$ and $y_7 = 0.2$ m.

Figures 1–4 demonstrate rather a good coincidence of theoretical and experimental results thus confirming the admissibility of the simplifying assumptions made and sufficient accuracy of the proposed calculation method.

Some difference in the experimental results for the concentration (Fig. 3), i.e. the decrease at small times and increase at large times, seems to be due to the effect of gravity. This conclusion is confirmed by the predicted dependence of the mixture density on the height y given in Fig. 5. Curves 1, 2 and 3 correspond to the time instants $t_1 = 60$, $t_2 = 300$ and $t_3 = 600$ s.

It follows from the results obtained that there are large temperature and concentration gradients in the vicinity of the evaporation surface. Substantial variations in the parameters occur in a comparatively narrow zone near the phase interface. This result

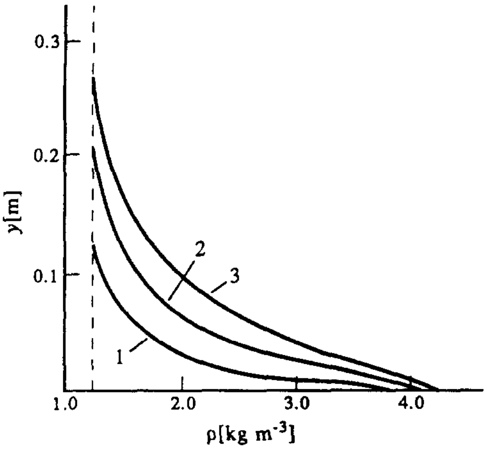

FIG. 5. Dependence of mixture density on the height above the evaporation surface.

agrees qualitatively with the results of practically observed evaporations of condensed gases into the atmosphere.

The solutions obtained most accurately describe the process near the phase change surface over limited time intervals being a case of practical interest.

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